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DELAWARE ESTUARY COMPREHENSIVE STUDY. REPORT NUMBER 1. REPORT ON ANALYSIS OF THE DYE DIFFUSION DATA IN THE DELAWARE RIVER ESTUARY. EVALUATION OF DIFFUSION COEFFICIENTS

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Delaware Estuary Comprehensive Study

Report Number 1

Division of Water Supply and Pollution Control
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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service

DELAWARE ESTUARY COMPREHENSIVE STUDY

TECHNICAL REPORT NO. 1

REPORT ON
ANALYSIS OF THE DYE DIFFUSION DATA IN THE DELAWARE RIVER ESTUARY

EVALUATION OF DIFFUSION COEFFICIENTS

Submitted to

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United States Public Health Service
Philadelphia, Pa.

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INTRODUCTION

This report presents an analysis of data collected in various experiments in the model of the Delaware River Estuary. Over the past ten years, a series of tests were conducted by various agencies in this model, which is located at the Corps of Engineers Station in Vicksburg, Mississippi. Generally, it was the purpose of these studies to measure the dispersion characteristics and transit time for various fresh-water discharges. The previous work was directed to an evaluation of salinity intrusion, pollution capacity and water quality. The most recent work in this regard was reported in "Dispersion Studies on the Delaware River Estuary Model and Potential Applications Toward Stream Purification Capacity Evaluations" sponsored by the Interstate Commission on the Delaware River Basin, Delaware State Water Pollution Commission, New Jersey Department of Health, Pennsylvania Department of Health, Water Department - City of Philadelphia, and the U. S. Public Health Service, Region II. In the report, which was issued in June 1961, considerable data was presented on both the instantaneous and continuous release experiments. It is the purpose of this report to analyze these data by means of the classical diffusion equation, to evaluate the diffusion coefficient for the various test conditions and to investigate the effect of various hydraulic and physical parameters on the coefficients.

THEORETICAL CONSIDERATIONS

When a soluble substance is discharged into an estuary, its motion toward the sea results from advection by the fresh-water flow and diffusion due to the tidal motion. Spatial and temporal concentration profiles of these substances are usually described by the classical diffusion equation. If, in addition, the substance is non-conservative, reaction effects must also be included in the basic equation. Vertical and lateral uniformity as observed in the Delaware River reduces the problem to a one-dimensional case. The differential equation, which describes the time rate of change of the concentration along the longitudinal axis for constant fresh-water discharge is:

$$\frac{\partial c}{\partial t} = \frac{1}{A}\frac{\partial}{\partial x}\left(EA\frac{\partial c}{\partial x}\right) - U\frac{\partial c}{\partial x} - Kc \qquad (1)$$

The first term on the right-hand side of Equation (1) is the diffusive effect including both a variable coefficient and cross-sectional area. The second term is the advection; the coefficient, U, varies with distance, due to changing cross-sectional area. The last term allows for a reactive effect, assuming first order kinetics. Equation (1) describes the time rate of change of concentration of a non-conservative substance in the longitudinal direction, with variable area and diffusivity. Equation (1) contains no term for the tidal velocity since the net effect of this factor on advection is negligible. Assume a mass of material is introduced at some

location in the estuary at high water slack. It moves seaward on the ebb tide and back on the subsequent flood. The material is dispersed by turbulent mixing and reduced by the reaction, but if there is no fresh-water advection, the center of mass would return to the point of injection. The time scale for the rate of change is therefore tidal cycles. The analysis is made from cycle to cycle at the high or the low-water slack period. The experimental data were collected under similar conditions.

Instantaneous Source

As an initial step in the analysis of estuarine data, consider the case of a constant cross-sectional area, velocity and diffusion. It is realized that this condition is an approximation; but, from an engineering point of view, it is appropriate because it permits determination of the order of magnitude of the coefficients. The coefficients, thus evaluated, may be functionally related to distance. These relationships may then be inserted in Equation (1) and a more exact solution sought. With constant coefficients, Equation (1) reduces to:

$$\frac{\partial c}{\partial t} = E\frac{\partial^2 c}{\partial x^2} - U\frac{\partial c}{\partial x} - Kc \qquad (2)$$

The solution of Equation (2) for an instantaneous release of a mass of material, M, which satisfies the boundary condition that $c = 0$ when $t = \infty$ is:

$$c = \frac{M}{2A\sqrt{\pi E t}}\, e^{\left[-\frac{(x-Ut)^2}{4Et} - Kt\right]} \qquad (3)$$

it may be seen that the spatial distributions are symmetrical for a specific time or tidal cycle. The peak concentrations of the spatial distributions occur at x = Ut and the dispersion about the peak may be described by:

$$c = c_o e^{\left[-\frac{x_1^2}{4Et} - Kt\right]} = c_p e^{\left[-\frac{x_1^2}{4Et}\right]} \qquad (4)$$

in which $c_o = \frac{M}{2A\sqrt{\pi E t}}$ = peak concentration at time, t, of a conservative substance

$c_p = c_o e^{-Kt}$ = peak concentration at time, t, of the non-conservative substance

X_1 = distance measured from the peak concentration

These equations may be used to describe the spatial distribution about a point which moves with the mean velocity of flow.

The temporal distributions, on the other hand, for a fixed point in space are asymmetrical and the degree of skewness depends upon the magnitude of Ut as compared to $(4Et)^{\frac{1}{2}}$.

Continuous Source

If a substance in solution or suspension is released continuously, its concentration will increase in time at a fixed location until an equilibrium value is achieved. The manner in which this increase occurs for a conservative substance may be

determined at a particular station by integrating Equation (3) with K = 0 from $t=0$ to $t=t_1$, as follows (Carslaw & Jaegar, 1960).

$$c = \frac{W}{2A\sqrt{\pi E}} \int_0^{t_1} \frac{1}{\sqrt{t-t_1}} e^{\left[-\frac{x^2}{4E(t-t_1)}\right]} dt_1 \tag{5}$$

This equation gives the concentration as a function of the time which has elapsed since the start of the continuous source, limiting at the mass rate, W. Equation (5) applies to a conservative substance with no advective term. For a constant rate of discharge it integrates to:

$$c = W\sqrt{\frac{t}{\pi E}}\, e^{\left[-\frac{x^2}{4Et}\right]} - \frac{Wx}{2E} \operatorname{erfc} \frac{x}{2\sqrt{Et}} \tag{6}$$

Equation (5) describes the buildup of concentration in time at a fixed location or the spatial profile for a given time. When the advection term is included, Equation (5) becomes:

$$c = \frac{W}{2A\sqrt{\pi E}} \int_0^{t} \frac{1}{\sqrt{t-t_1}} e^{\left[-\frac{[x-U(t-t_1)]^2}{4E(t-t_1)}\right]} dt_1 \tag{7}$$

Equation (7) cannot be integrated directly and a numerical solution may be obtained, using the following equation:

$$c = \frac{2W}{AU\sqrt{\pi}} e^{\frac{Ux}{2E}} \int_0^{\omega} e^{-\left[\omega^2 + \frac{\theta^2}{\omega^2}\right]} d\omega \tag{8}$$

in which $\theta = \frac{xU}{4E}$

$$\omega = \frac{U}{2}\sqrt{\frac{t}{E}}$$

If a reaction term is also included then Equation (8) becomes:

$$c = \frac{Wx}{2AE\sqrt{\pi\theta}} e^{\frac{Ux}{2E}} \int_0^{\omega} e^{-\left[\omega^2 + \frac{\theta^2}{\omega^2}\right]} d\omega$$

$$\theta = \frac{x}{4E}\sqrt{U^2 + 4KE}$$

$$\omega = \frac{\sqrt{4E\theta t}}{x} \tag{9}$$

The steady-state solution of Equation (7) is:

$$c = \frac{W}{AU} \qquad x > 0 \tag{10}$$

$$c = \frac{W}{AU} e^{\frac{Ux}{E}} \qquad x < 0 \tag{11}$$

and for Equation (8)

$$C = \frac{W}{A\sqrt{U^2 + 4KE}}\, e^{\frac{Ux}{2E}\left[1 - \sqrt{1 + \frac{4KE}{U^2}}\right]} \qquad (12)$$

The preceding equations are useful in predicting equilibrium concentration and the time required to achieve the equilibrium condition for non-conservative substances.

Analysis of Model Data

Experimental work in the model of the Delaware River Estuary provide excellent data on dispersion patterns of salinity and of soluable material released on various locations in the estuary. A map of the Delaware Estuary, showing the stations and the limits of the models, is presented in Figure 1. Some of the experimental difficulties described in the earlier work were minimized in a subsequent set of tests recently reported (Joint Study Committee, 1961). In these experiments dye was released as a plane source at various locations for different flow conditions. Both instantaneous and continuous source experiments were conducted. The concentrations of dye at many stations and over many tidal cycles were measured. In all of the experiments reported, dye was adsorbed on the surface of the model. In both of the previous analyses (Chesapeake Bay Institute, 1958 and Joint Study Committee, 1961) the observed concentrations were adjusted to compensate for the loss. In this work, however, the loss was incorporated in original differential equation as a reaction term and specifically in Equation (2) as a first order reaction.

The rate of loss of mass may be written:

$$\frac{dM}{dt} = -KM \tag{13}$$

which integrates to:

$$M = M_o e^{-Kt} \tag{14}$$

That the dye absorption may be described by first-order kinetics was arrived at by the following analysis: the instantaneous release tests produced the typical dispersion patterns for the various tidal cycles, as shown in Figure 8. The total mass in solution for various cycles was termined by taking mean concentration values from the spatial distributions for each 10,000 ft. interval and multiplying by the volume within that interval. The sum of these incremental masses gave the total mass present for each cycle. A plot of these data are shown in Figure 2. in which the total mass remaining is plotted against tidal cycles. The initial mass of dye released in each test was 2.6 grams. Apparently in test 10 there was a relatively high initial adsorption, since the zero intercept is approximately 2.1 grams. Expressing the mass as a percent of the intial quantity, the data are re-plotted in Figure 2 on linear coordinates, employing an initial mass of 2.6 grams for tests 11 and 12 and 2.1 grams for test 10. A reaction coefficient of 0.022 per cycle was employed to calculate the line shown in this figure. Since equation (14) does not take into account the distribution of the concentration within the associated volume the reaction coefficient, determined by

means of this equation for the mass, simply yields an approximate value for the concentration.

The first step in the analysis of the instantaneous release data consisted in selecting an equation, and a method, which would most closely approximate the condition of variable coefficients. The temporal distributions at various stations are appropriate in this regard. The coefficients which define this distribution could then be applied to the selected station. Re-arranging Equation (3), we have

$$\ln c\sqrt{t}\, e^{+Kt} - \ln B = -\frac{(x-Ut)^2}{4Et} \tag{15}$$

in which $B = \frac{M}{2A\sqrt{\pi}}$ = constant for a particular station. Equation (15) linearizes the relationship and a plot of the logarithm of $c\sqrt{t}\, e^{+Kt}$ versus $\frac{(x-Ut)^2}{4t}$ yields a straight line, whose slope is a measure of E. The velocity U, is calculated from the continuity equation, employing the cross-sectional area at the station. Typical examples are presented in Figure 3. The intercept permits evaluation of the term B. This method was applied to a series of stations for the various tests and the diffusion coefficient was evaluated as described.

It is significant to compare the results obtained from an analysis of the temporal distribution with those obtained from the spatial distributions. It is appreciated that the latter method is more approximate, since the coefficients vary with distance. In spite of this obvious limitation, Equation (4) has certain advantages since

both the advection and reaction coefficients are eliminated, i.e. the data are analyzed with respect to the peak concentration for a specific tidal cycle. The procedure consisted in assigning the spatial location of the peak concentration. The distances from this point to the stations at which concentrations were measured were then calculated. The logarithm of the concentration was plotted against an arithmetic scale of the distance squared. Examples are shown in Figure 4. The concentrations upstream and downstream from the peak for various tidal cycles were plotted and the diffusion coefficient was determined from the slope of the line. The coefficient thus calculated was assigned to the station at which the peak concentration occurred. Since the peak concentration of the spatial distributions occur at $x = Ut$, it was then possible to calculate the value of the velocity, U, which would be consistent with the above step. The velocity thus calculated agreed reasonably well with that calculated by dividing the fresh-water discharge by the area at the location of the peak.

At this point it was possible to assign, at least an approximate, if not an exact, value of each of the three coefficients. The release point for tests 10, 11 and 12 was at station 0 + 00. The fresh-water discharge from this point to the Schuylkill River is the sum of the Delaware River flow at Trenton and Rancocas Creek. Downstream from the Schuylkill, the total fresh-water in Delaware was increased by the amount of flow from this tributary. For a specific station, the diffusion and advection may be assigned. The reaction

coefficient was assumed to be the same as that for the mass analysis (K = 0.022 per cycle). The temporal distributions were then calculated by means of Equation (3). The area term in the denominator of Equation (3) is that at the station under consideration. Examples of these distributions and the observed data at various stations are shown in Figure 5 for test 11. (Similar plots for tests 10 and 12 in Figure 6). The temporal distributions for a given station and the three different fresh-water flow conditions are presented in Figure 7. Typical spatial distributions are shown in Figure 8. The diffusion and advection coefficients used in the calculation are those associated with the station at which the peak concentration occurs. A summary of the data employed in these analyses is shown in Table 1.

The analysis of the data of the continuous release experiments were more limited, although they equally lend themselves to similar treatment. However, an additional parameter, which must be assigned, is the time at which the dye adsorption may be assumed to cease at a given location. The adsorption capacity of the model surface is apparently reached during the course of the continuous experiments. This seems particularly apparent in the reaches just downstream from the release point, i.e. the equilibrium concentrations at the point of release and the next two stations downstream are equal, indicating no loss in this reach. The time at which adsorption may be assumed to cease increases in downstream direction, because of the increased surface area. An example of the continuous release data is shown in Figure 9. The time at which the dye adsorption capacity was reached delineated the periods over which Equations (8)

and (9) were respectively used. From zero up to this time, Equation (9) was employed to calculate the concentration build-up as shown in Figure 9 and Equation (8) was used for times greater than this.

CONCLUSIONS

The agreement between the observed data and calculated curves for the various test conditions is reasonably good for both the temporal and spatial distributions. It is particularly significant that a consistent set of diffusion, advection and reaction coefficients were used for both continuous and instantaneous release conditions. It is seen that the diffusion coefficient decreases with distance; no distinct pattern is evident from the fresh-water flow. Future effort should be directed toward the evaluation of the nature of these relationships. The functional relationships between distance and, respectively, velocity and area should be incorporated into the original differential equation. Solutions of this equation, with the variable coefficients should be sought. It is strongly recommended that analog computation be employed for solution of these equations.

NOTATIONS

A	Cross-sectional area
B	Constant (Equation 15)
c	Concentration
c_o	Peak concentration of conservative substance
c_p	Peak concentration of non-conservative substance
E	Diffusion coefficient
K	Reaction coefficient
M	Mass
t	Time
U	Advection coefficient
W	Rate of continuous release
θ	Parameter - Equations (8) and (9)
ω	Parameter - Equations (8) and (9)
erfc	Complimentary error function

TABLE 1

DIFFUSION COEFFICIENTS FOR VARIOUS TESTS

TEST 10 Q = 3400 CFS* K = 0.022/C		TEST 11 Q = 7000 K = 0.022		TEST 12 Q = 12350 K = 0.022/C		TEST 13 Q = 3400 K = 0.027		TEST 14 Q = 5500 K = 0.018	
Station	E(SMPC)	Station	E	Station	E	Station	E	Station	E
0	1.6	0	2.6	0	2.3	-145	1.0	-145	1.9
20	2.1	20	2.3	20	2.3	-135	0.5	-135	0.9
40	1.3	40	1.0	50	0.9	-125	0.8	-125	0.9
60	1.0	60	2.0	60	1.1	-115	0.9	-95	3.1
100	1.3	80	1.2	80	1.4	-95	1.8	-60	1.1
		100	1.0	100	1.6	-75	1.3	-25	1.3
						-55	0.5		
						-40	0.7		

*NOTE: ALL FLOWS MEASURED AT TRENTON

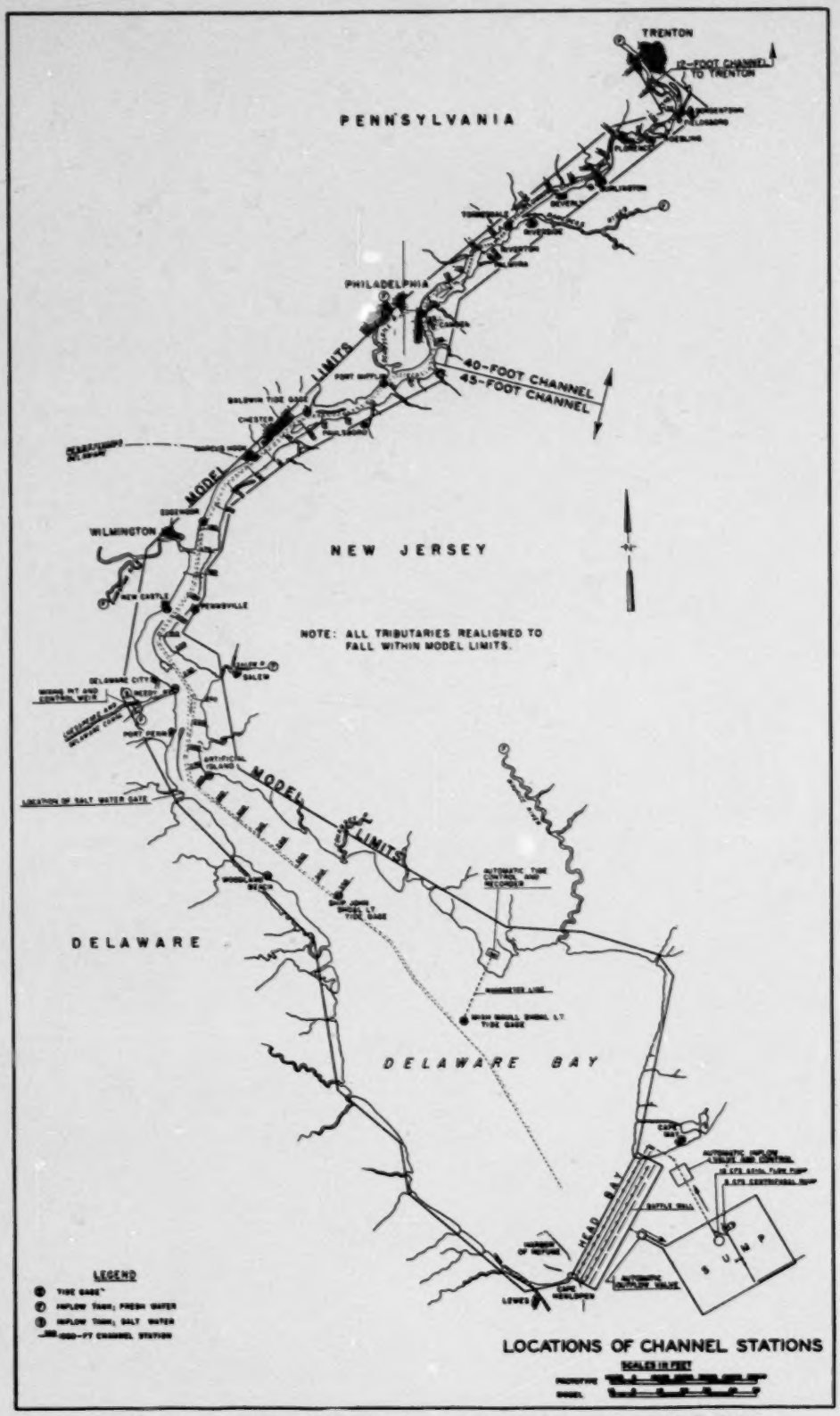

FIG. 1 – DELAWARE RIVER ESTUARY LOCATION MAP

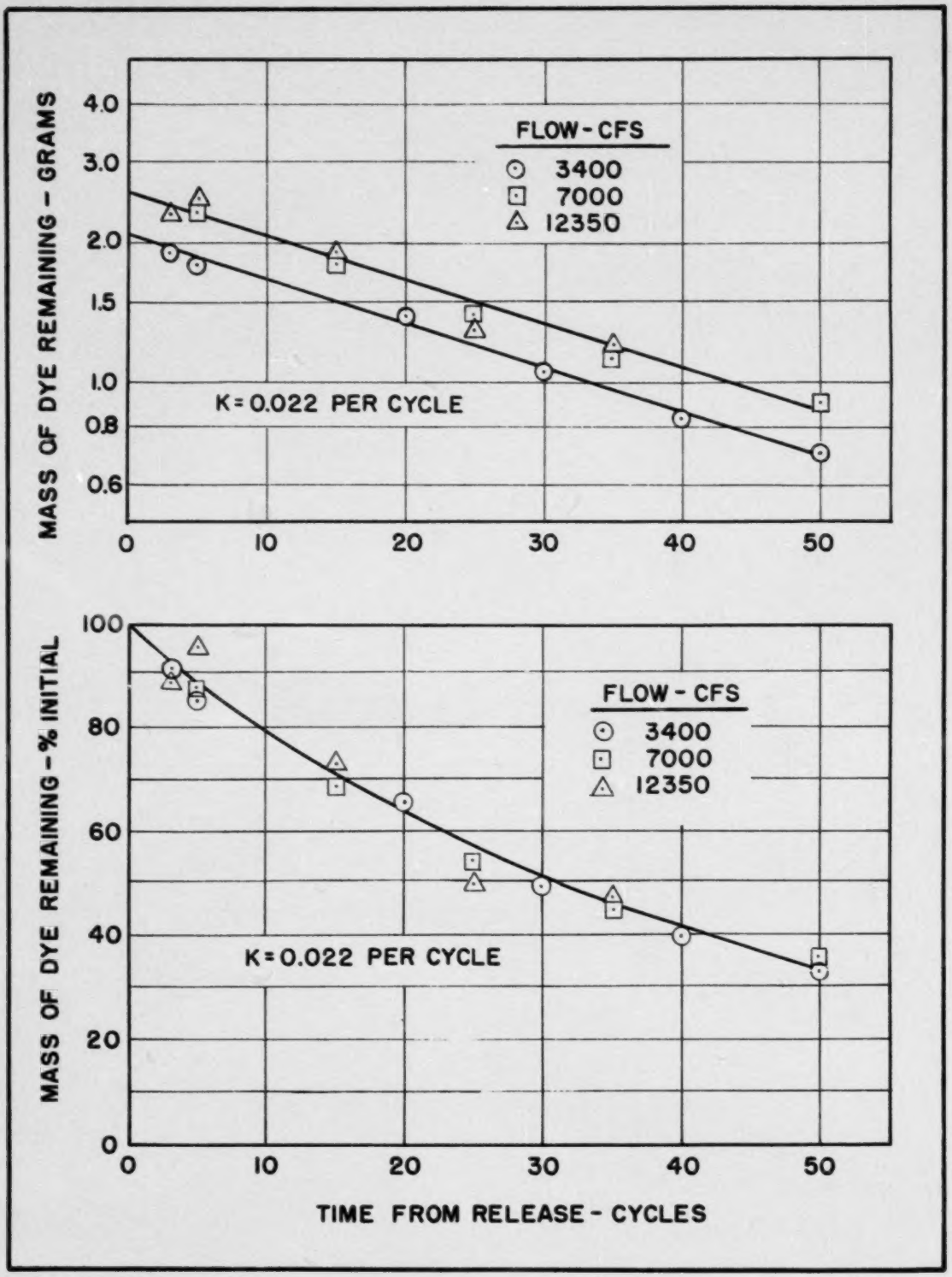

FIG. 2 - DYE LOSS

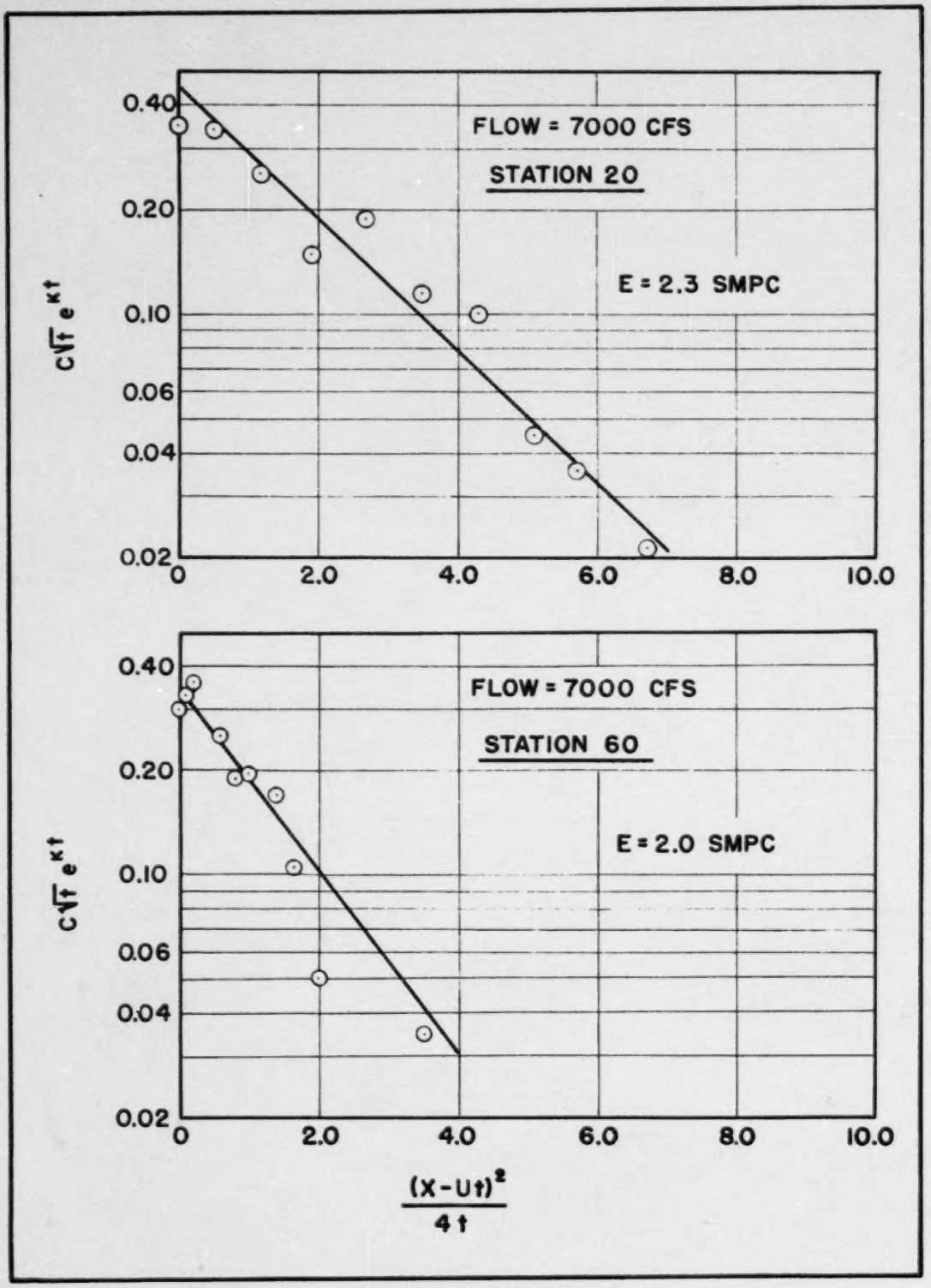

FIG. 3 - ANALYSIS OF DATA - TEMPORAL DISTRIBUTIONS

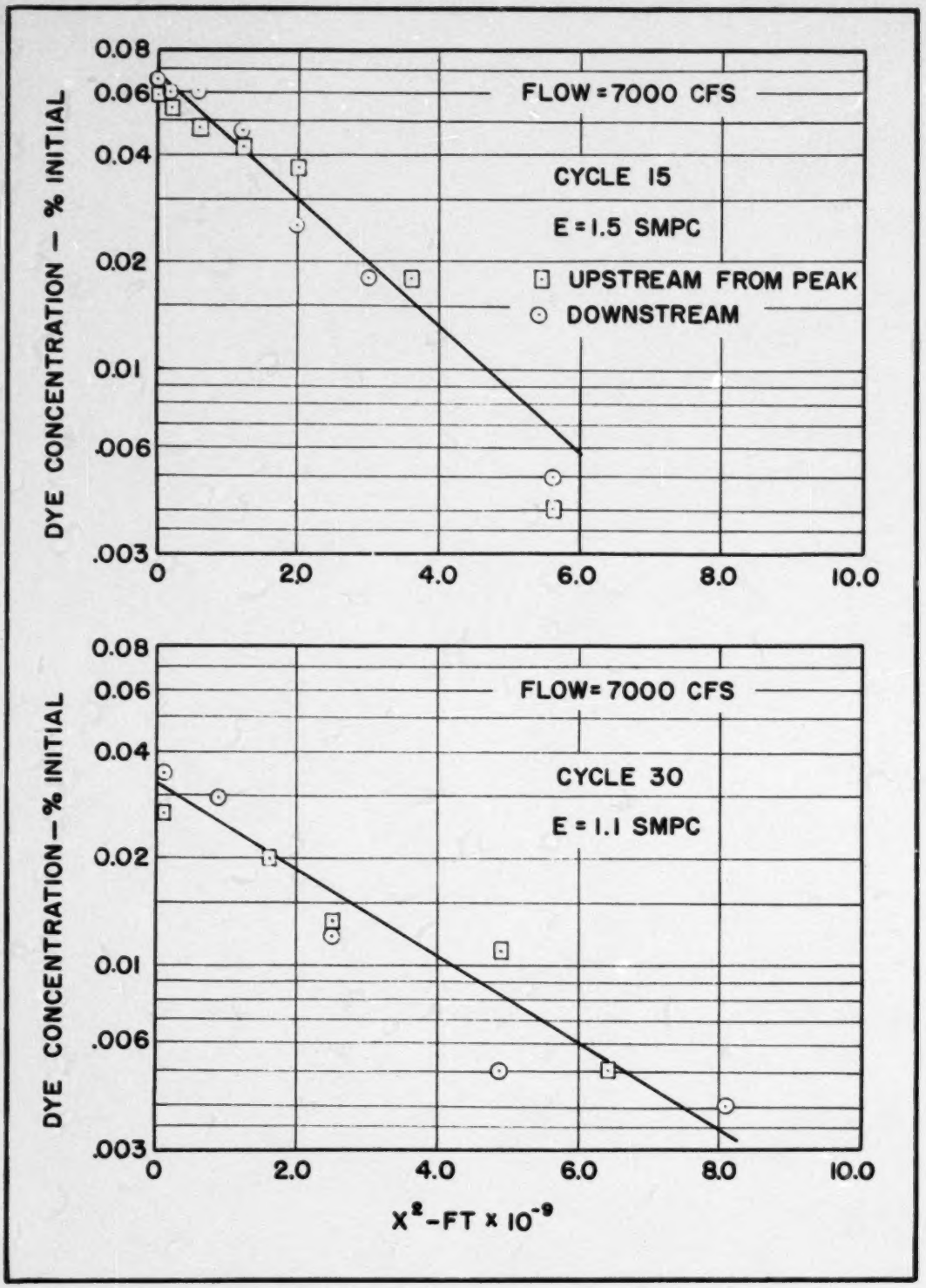

FIG. 4 - ANALYSIS OF DATA - SPATIAL DISTRIBUTIONS

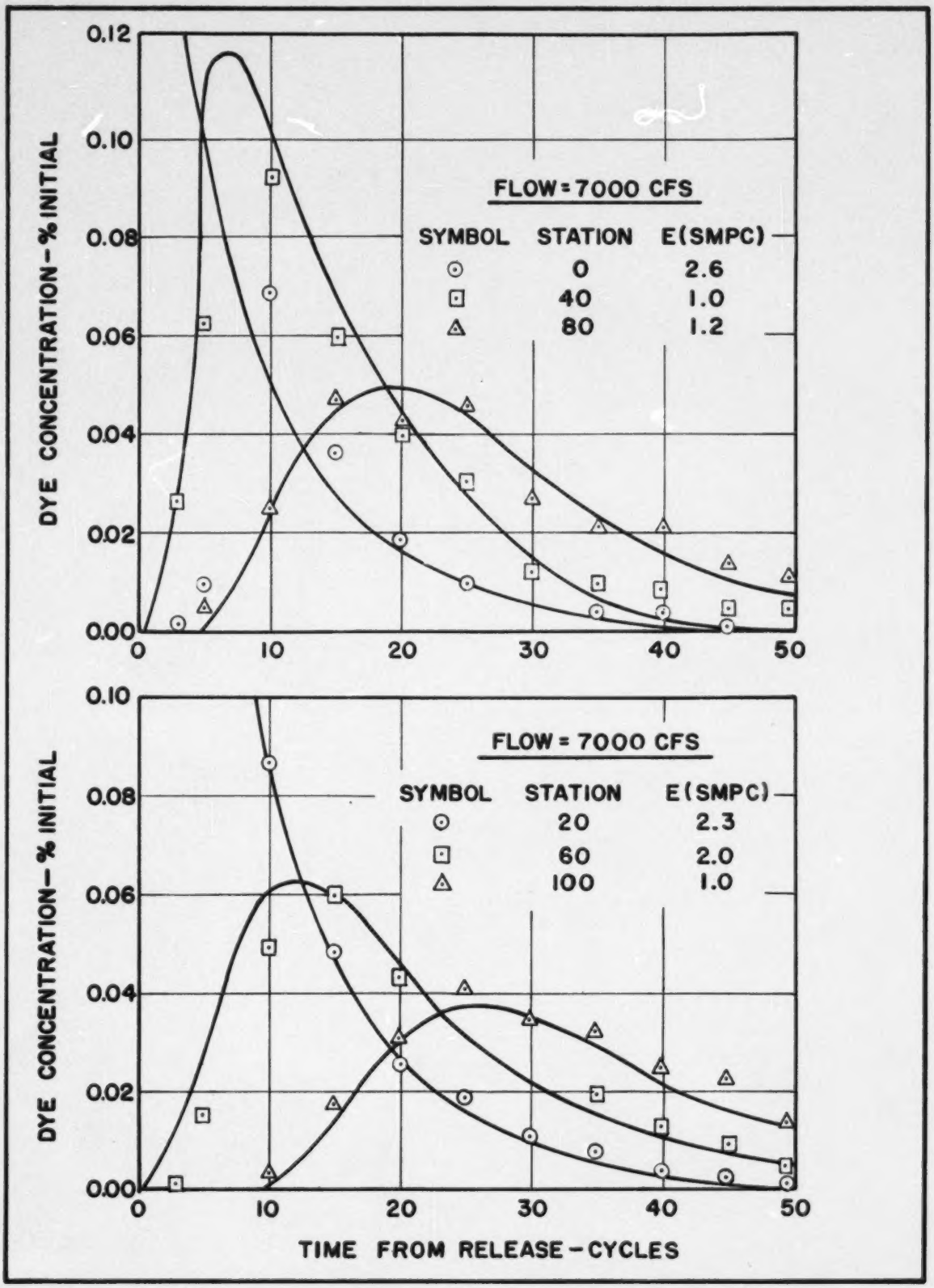

FIG. 5 - TEMPORAL DISTRIBUTIONS

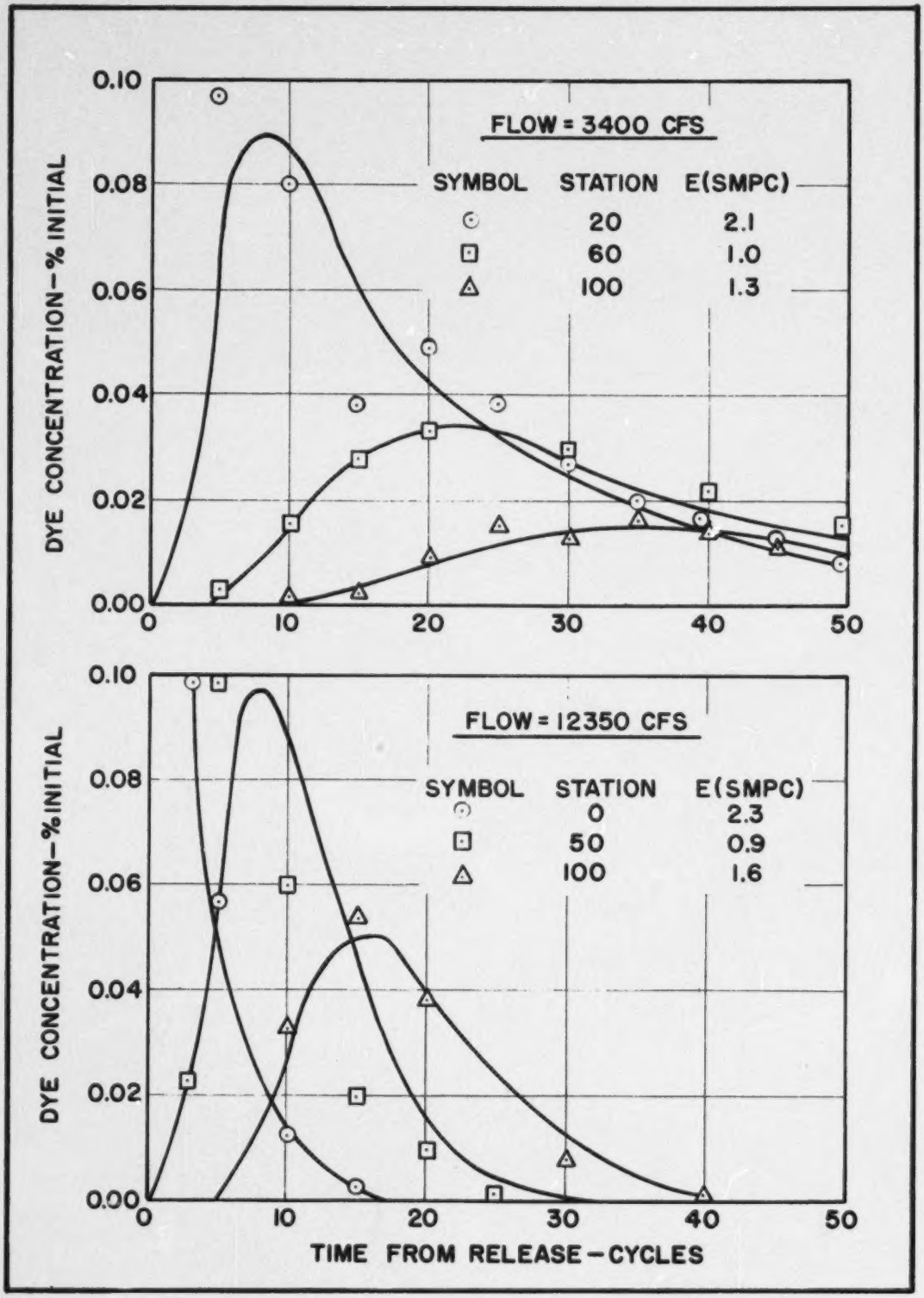

FIG. 6- TEMPORAL DISTRIBUTIONS

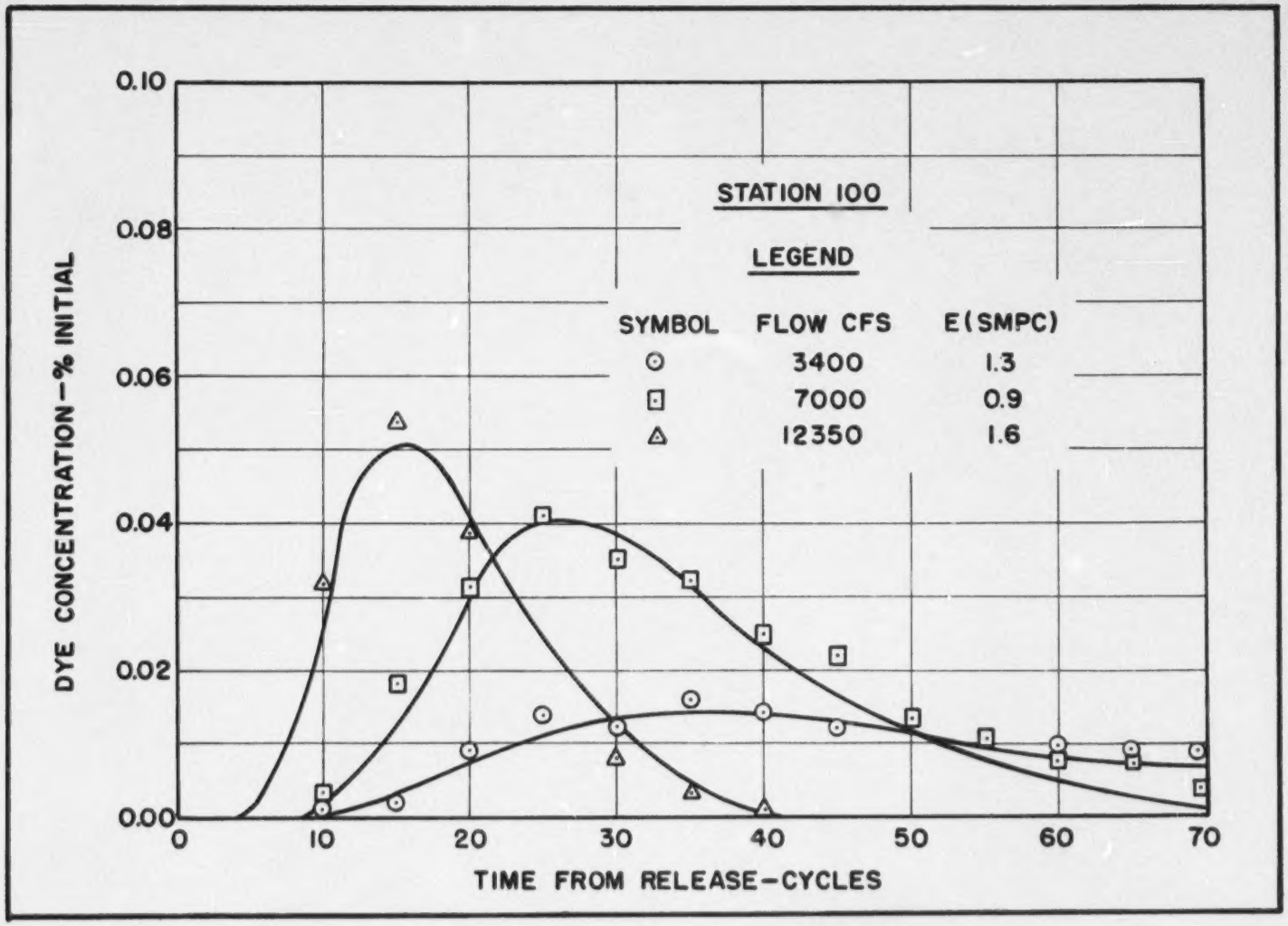

FIG. 7 - TEMPORAL DISTRIBUTIONS

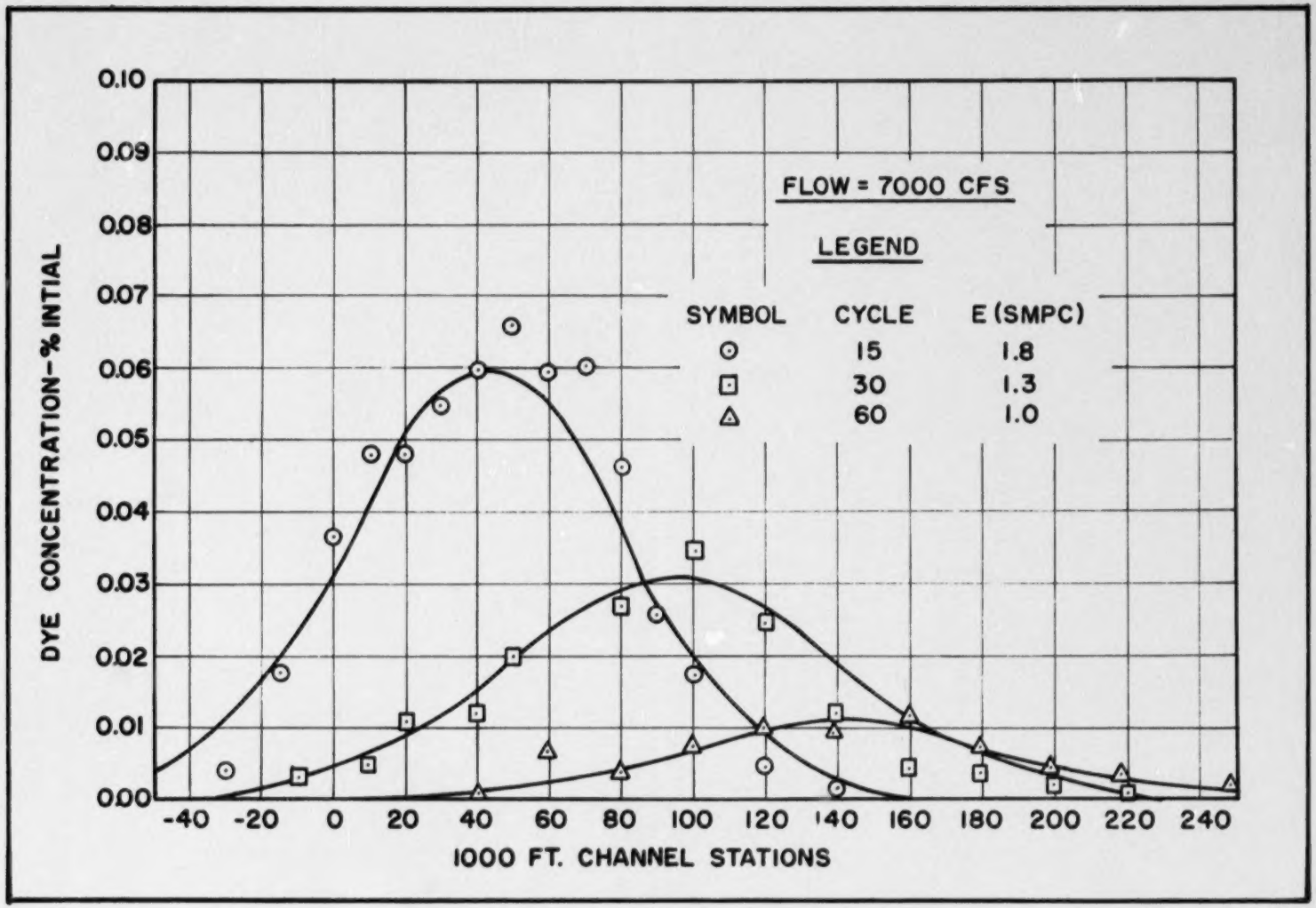

FIG. 8 - SPATIAL DISTRIBUTIONS

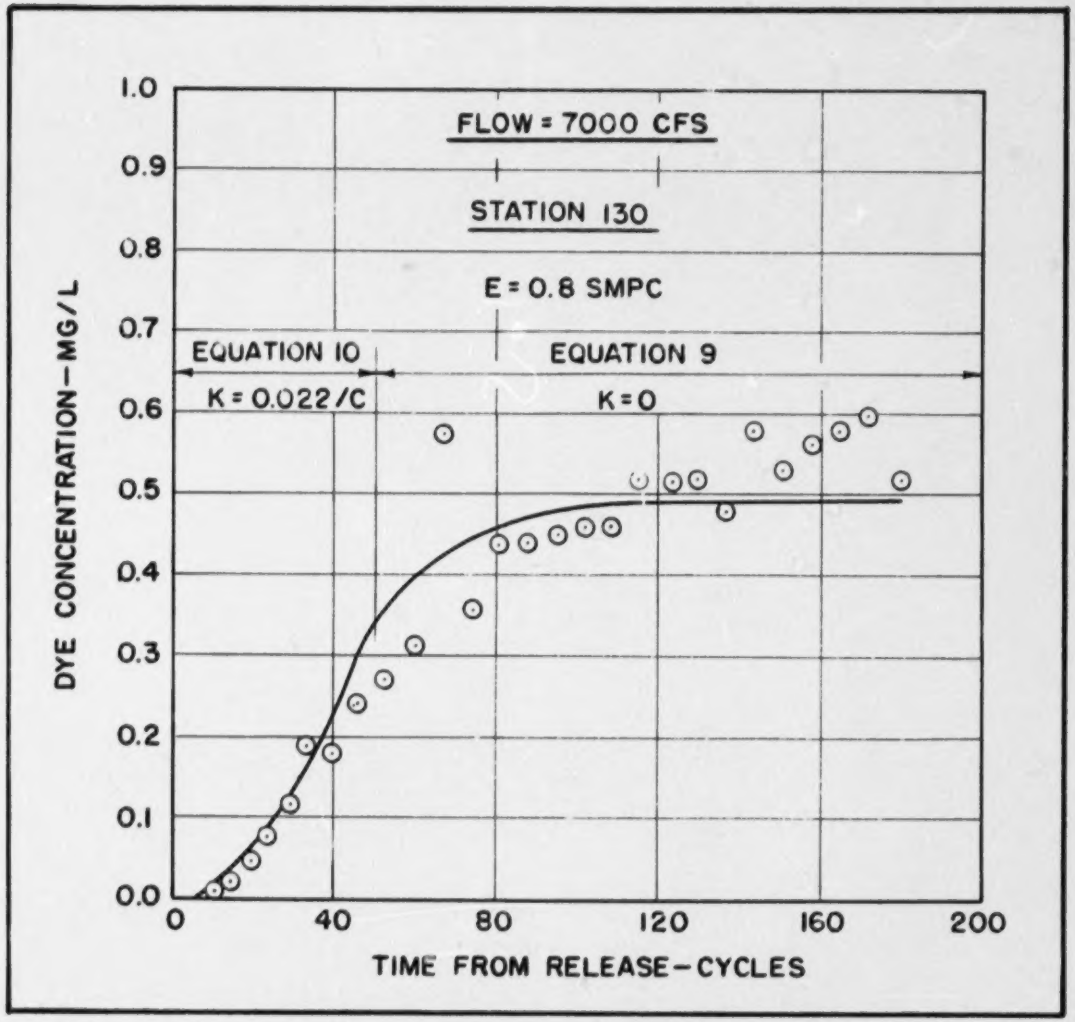

FIG. 9 - TEMPORAL DISTRIBUTIONS FOR CONTINUOUS RELEASE

GPO 946-524